Math 417: Homework 1

Due Friday, September 1, 2023

1. Goodman, exercise 1.3.1. In this problem the only symmetries considered are rotations (not reflections).

2. Goodman, exercise 1.3.2.

3. Goodman, exercise 1.4.2. The last sentence ("Verify that…") implies that one should verify all 36 possible products of the 6 matrices. For this homework, please verify at least three matrix products that do not involve an identity matrix.

4. Let X be a finite set. Suppose $f : X \to X$ is any map. Show that f is injective (i.e., one-to-one) if and only if f is surjective (i.e., onto).

5. Goodman, exercise 1.5.1.

6. Goodman, exercise 1.5.2.

7. Goodman, exercise 1.5.8.

8. Let $k \geq 2$ be a natural number. Let $\sigma \in S_n$ be a k-cycle. Prove that $\sigma^k = e$, while $\sigma^\ell \neq e$ for any $1 \leq \ell < k$. (That is, show that σ has *order* k. This fact is asserted without proof on page 20 of the textbook. The purpose of this exercise is to prove it.)